Girraween High School

Year 12 Mathematics Extension 1 2026

Task 2 – Half Yearly Examination

Student Number _____

Question	Mark	ME12-1	ME12-2	ME12-3	MA12-7
Section 1					
1	/1				✓
2	/1				✓
3	/1		✓		✓
4	/1	✓			✓
5	/1			✓	✓
Section 2					
6	/11				✓
7	/13			✓	✓
8	/9		✓		✓
9 a	/4	✓			✓
b	/6				✓
c	/3		✓		✓
10 a	/3		✓		✓
b	/5	✓			✓
Q11 a	/4			✓	✓
b	/5		✓		✓
Total	/68	/10	/21	/18	/68

Year 12 Mathematics Extension 1 outcomes

A student:

ME12-1 applies techniques involving proof or calculus to model and solve problems

ME12-2 applies concepts and techniques involving vectors and projectiles to solve problems

ME12-3 applies advanced concepts and techniques in simplifying expressions involving compound angles and solving trigonometric equations

ME12-7 evaluates and justifies conclusions, communicating a position clearly in appropriate mathematical forms

Year 12 Task 2
2026
HSC Mathematics Extension 1
Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ⬭ B ⬬ C ⬭ D ⬭

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ⬬ B ⬬ (crossed) C ⬭ D ⬭

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

Correct ↓

A ⬬ (crossed) B ⬬ (crossed) C ⬭ D ⬭

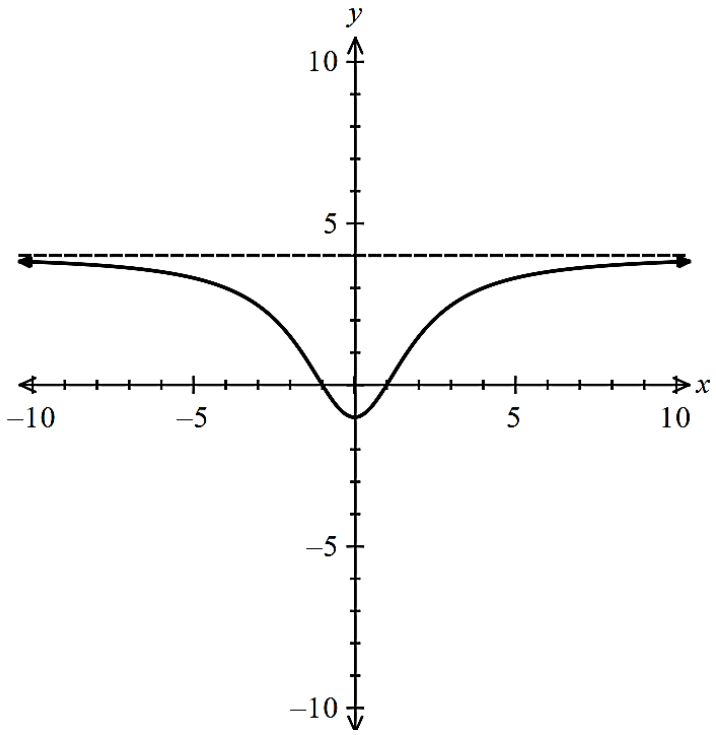
1.	A	○	B	○	C	○	D ○
2.	A	○	B	○	C	○	D ○
3.	A	○	B	○	C	○	D ○
4.	A	○	B	○	C	○	D ○
5.	A	○	B	○	C	○	D ○

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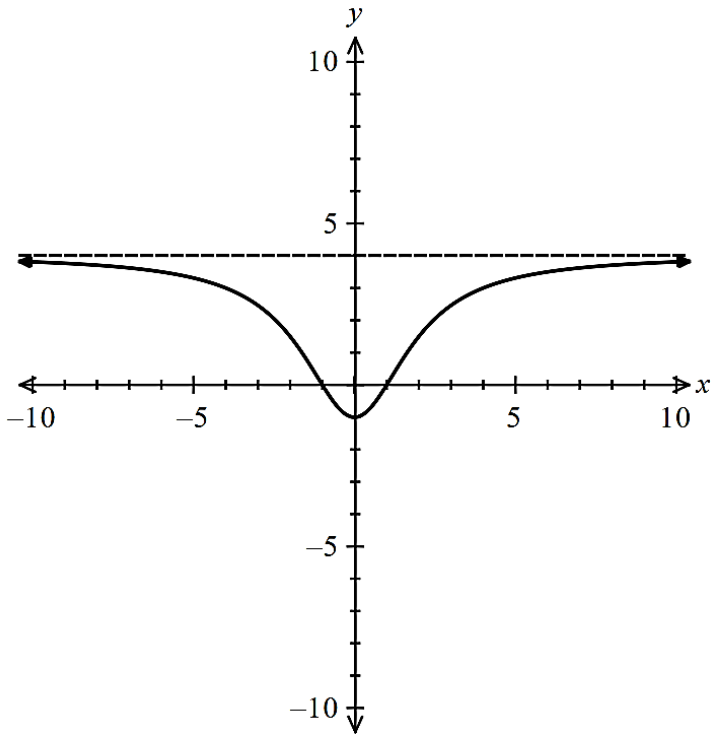
Answer Sheet for Question 9 (b)

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i) **2**

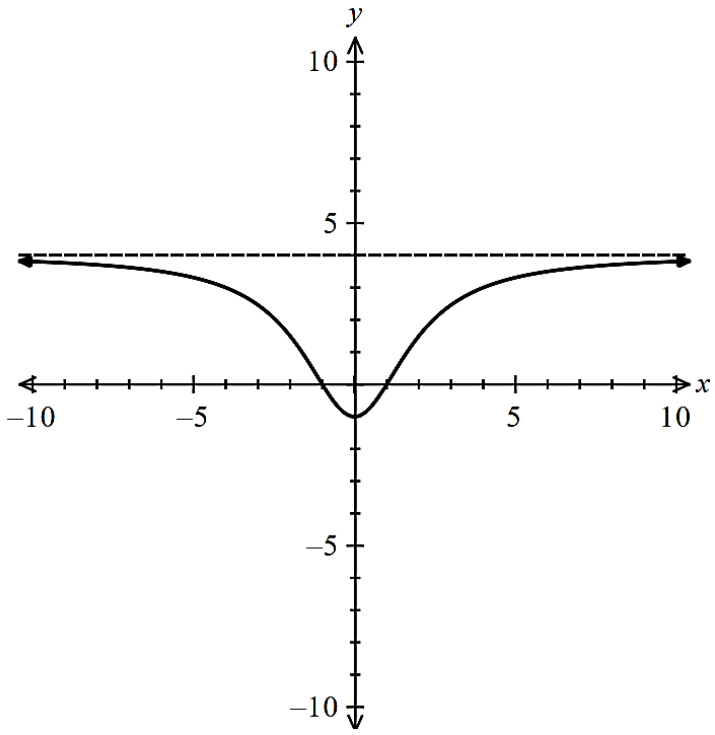


ii) **2**



Question 9 (b) continues over page

iii) **2**



End of Question 9 (b)

Girraween High School

2026

Year 12 HSC Task 2

Mathematics Extension 1

General Instructions

- Reading time: 5 minutes
- Working time: 90 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Answer multiple-choice questions on the multiple-choice answer sheet provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Question 9 (b) has a separate answer sheet with axes for graphs

Total Marks: 68

Section I

Multiple Choice (5 Marks)

- Attempt Questions 1 – 5
- Allow about 10 minutes for this section

Section II

Extended Response (63 Marks)

- Attempt Questions 6 – 11
- Allow about 80 minutes for this section

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Section I: Multiple Choice

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 5.

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1. Which of the following is equivalent to the expression $\dfrac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1}$?

(A) $\cot\theta$ (B) $\sec\theta$

(C) $\sin\theta$ (D) $\tan\theta$

2. A curve has parametric equations $x = \dfrac{2}{t}$ and $y = 2t^2$.

What is the Cartesian equation of this curve?

(A) $y = \dfrac{4}{x}$ (B) $y = \dfrac{8}{x}$

(C) $y = \dfrac{4}{x^2}$ (D) $y = \dfrac{8}{x^2}$

3. Given that $\underset{\sim}{u} = 5\underset{\sim}{i} - 12\underset{\sim}{j}$ and $\underset{\sim}{v} = 4\underset{\sim}{i} + 3\underset{\sim}{j}$, find $\text{proj}_{\boldsymbol{u}}\,\boldsymbol{v}$.

(A) $-\dfrac{80}{169}\underset{\sim}{i} + \dfrac{192}{169}\underset{\sim}{j}$ (B) $-\dfrac{64}{25}\underset{\sim}{i} - \dfrac{48}{25}\underset{\sim}{j}$

(C) $-\dfrac{16}{169}\left(4\underset{\sim}{i} + 3\underset{\sim}{j}\right)$ (D) $\dfrac{16}{25}\left(5\underset{\sim}{i} - 12\underset{\sim}{j}\right)$

Section 1 continues over page

4. Victoria made an error proving that $3^{2n}-1$ is divisible by 8 (where n is an integer greater than 0) using mathematical induction. Part of the proof is shown below:

Step 2: Assume the result true for $n=k$

$3^{2k}-1=8P$ where P is an integer. Line 1

Hence $3^{2k}=8P+1$

Step 3: To prove the result is true for $n=k+1$

$3^{2(k+1)}-1=8Q$ where Q is an integer. Line 2

$$\begin{aligned}
\text{LHS} &= 3^{2(k+1)}-1 \\
&= 3^{2k}\times 3^2-1 \\
&= (8P+1)\times 3^2-1 \quad \text{Line 3} \\
&= 72P+1-1 \quad \text{Line 4} \\
&= 72P \\
&= 8(9P) \\
&= 8Q \\
&= \text{RHS}
\end{aligned}$$

Which line did Victoria make an error?

(A) Line 1 (B) Line 2

(C) Line 3 (D) Line 4

Section 1 continues over page

5. If $y = 2 \sin x - 4 \cos x$ is written in the form $y = R\cos(x - \alpha)$ where R > 0 and $0° \leq \alpha < 360°$, which of the following is approximately equal to α?

(A) $27°$

(B) $153°$

(C) $207°$

(D) $-27°$

End of Section I

Section II starts over page

Section II

63 Marks
Attempt Questions 6 - 11
Allow about 80 minutes for this section.

Answer each question on a **new page** in the writing booklet. Extra writing booklets are available. For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (11 marks) Start a new page in the writing booklet. **Marks**

a) Solve : $\dfrac{2x+5}{x+3} < 1$ **3**

b) In how many ways can a committee of 3 women and 4 men be selected from 8 women and 7 men, if Mr P refuses to be on the committee with Mrs Q? **2**

c) A polynomial $P(x) = x^4 + px^3 + qx^2 - 5x + 1$ has a double zero at $x = 1$. **3**
Find the values of p and q.

d) Consider the expansion of $(1 + x)(1 + x)^9$.

i) Write down the coefficient of x^r in this expansion. **2**

ii) Hence, or otherwise, prove that $\binom{10}{r} = \binom{9}{r-1} + \binom{9}{r}$ **1**

End of Question 6

Examination continues over page

Question 7 (13 Marks) Start a new page in the writing booklet.

a) Show that $4\cos 5\alpha \cos 2\alpha \sin \alpha = \sin 8\alpha - \sin 6\alpha + \sin 4\alpha - \sin 2\alpha$. **3**

b) Solve the following equation by letting $t = \tan\theta$. **4**

$$4\sin 2\theta - \tan\theta + 3\cos 2\theta = 3 \qquad \text{for } 0° \leq \theta \leq 360°$$

Give your answer(s) correct to the nearest degree where necessary.

c) i) Prove that $\cos 3x = 4\cos^3 x - 3\cos x$. **3**

ii) Hence, solve the equation $2\cos x = \sqrt{\cos 3x}$ for $0 \leq x \leq 2\pi$. **3**

End of Question 7

Examination continues over page

Question 8 (9 marks) Start a new page in the writing booklet.

a) Given that $\underset{\sim}{p} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\underset{\sim}{q} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, find:

i) $\left|\underset{\sim}{p} - \underset{\sim}{q}\right|$ **2**

ii) $\underset{\sim}{r}$ such that $\underset{\sim}{p} + 2\underset{\sim}{q} + \underset{\sim}{r} = 0$. **2**

b) If $\underset{\sim}{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} -5 \\ k \end{pmatrix}$, find the value of k given that:

i) $\underset{\sim}{u}$ is perpendicular to $\underset{\sim}{v}$ **1**

ii) $\underset{\sim}{v}$ is opposite in direction to $\underset{\sim}{u}$ **2**

iii) the origin and the points represented by the position vectors $\underset{\sim}{u}$, $\underset{\sim}{v}$ and $\underset{\sim}{u} + \underset{\sim}{v}$ form the vertices of a rhombus **2**

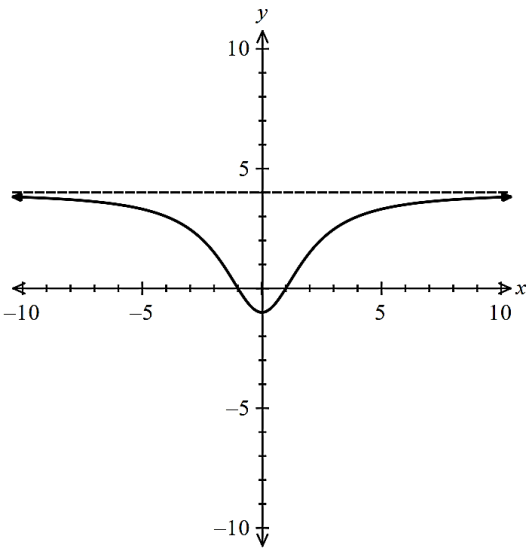
End of Question 8

Examination continues over page

Question 9 (13 marks) Start a new page in the writing booklet.

a) Prove by mathematical induction that $2^n - 1$ is divisible by 3, for all even positive integers. **4**

b) The function $y = f(x)$ shown has a horizontal asymptote at $y = 4$.



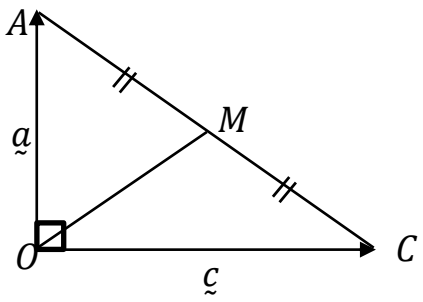
Using the graph above, sketch and label the following showing all important features such as intercepts and asymptotes. **Graph these on the axes provided on pages 5 and 6 in the Writing/Answer booklet.**

i) $|y| = f(x)$ **2**

ii) $y = \dfrac{1}{f(x)}$ **2**

iii) $y^2 = f(x)$ **2**

c) Let $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OC} = \underset{\sim}{c}$. Use vectors to prove that the midpoint of the hypotenuse of a right-angled triangle is equal in distance from all vertices of the triangle. **3**



End of Question 9

Examination continues over page

Question 10 (8 marks) Start a new page in the writing booklet.

a) The angle between vectors $a\underset{\sim}{i}+b\underset{\sim}{j}$ and $c\underset{\sim}{i}+d\underset{\sim}{j}$ is 45°, where a, b, c and d are real numbers. **3**

Show that $(ac + bd)^2 = (bc - ad)^2$.

b) i) Show that $(x^2+2x-1)(x+2) = x^3+4x^2+3x-2$ **1**

ii) Hence, prove by mathematical induction that, for all integers $n \geq 1$: **4**

$$0\times 1! + 7\times 2! + 26\times 3! + \ldots + (n^3-1)\times n! = (n^2-2)(n+1)! + 2$$

End of Question 10

Examination continues over page

Question 11 (9 marks) Start a new page in the writing booklet.

a) i) Simplify: $\dfrac{(2x\cos 2x)^3}{8\sin^3 2x}$ **1**

ii) Hence, or otherwise, evaluate: $\lim\limits_{x\to 0}\left(\dfrac{2x^3}{\tan^3 2x}\right)$ **3**

b) A triangle ABC has an area of 40 square units. $\angle ABC$ is acute. **5**

The vectors that form two of the sides are given by:

$$\overrightarrow{BA}=\begin{bmatrix}6\\2\end{bmatrix},\ \overrightarrow{BC}=\begin{bmatrix}p\\q\end{bmatrix} \text{ and } |\overrightarrow{BC}|=8\sqrt{5} \text{ units.}$$

Find the possible values of p and q.

End of Question 11

End of Assessment Task

Yr 12 Ext 1 - TASK 2 - SOLUTIONS 2026

& Feedback

Section 1: Multiple Choice

1/ $\dfrac{\sin 2\theta + \sin\theta}{\cos 2\theta + \cos\theta + 1} = \dfrac{2\sin\theta\cos\theta + \sin\theta}{2\cos^2\theta - 1 + \cos\theta + 1}$

$= \dfrac{\sin\theta(2\cos\theta + 1)}{\cos\theta(2\cos\theta + 1)}$

$= \tan\theta$

(D)

2/ $x = \dfrac{2}{t}, \; y = 2t^2$

$t = \dfrac{2}{x} \Rightarrow y = 2\times\left(\dfrac{2}{x}\right)^2$

$y = 2\times\dfrac{4}{x^2}$

$y = \dfrac{8}{x^2}$

(D)

3/ $\underline{u}\cdot\underline{v} = 5\times 4 + -12\times 3$ $\quad \underline{u} = 5\underline{i} - 12\underline{j}$

$= -16$ $\quad \underline{v} = 4\underline{i} + 3\underline{j}$

$|\underline{u}| = \sqrt{5^2 + (-12)^2}$

$= 13$

$\text{proj}_{\underline{u}}\underline{v} = \dfrac{\underline{u}\cdot\underline{v}}{|u|^2}\underline{u}$

$= \dfrac{-16}{13^2}(5\underline{i} - 12\underline{j})$

$= -\dfrac{80}{169}\underline{i} + \dfrac{192}{169}\underline{j}$

(A)

4/ Line 4 is incorrect.
Should read $72P+9-1$

(D)

5/ $R\cos(x-\alpha) = R\cos x\cos\alpha + R\sin x\sin\alpha$

$2\sin x - 4\cos x = R\cos\alpha\cos x + R\sin\alpha\sin x$

Equating

$-4 = R\cos\alpha$ ①

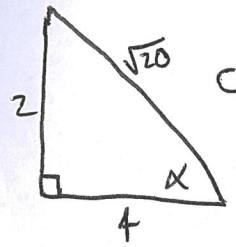
$2 = R\sin\alpha$ ②

Square and Add ① + ②

$20 = R^2(\cos^2\alpha + \sin^2\alpha)$

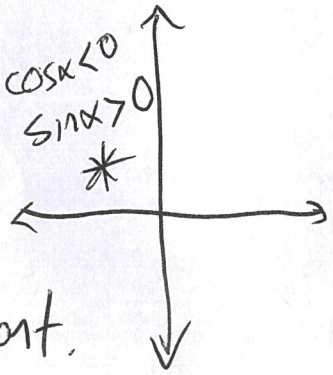
$\sqrt{20} = R$, $R > 0$

Now



$\cos\alpha = \frac{-4}{\sqrt{20}}$

$\sin\alpha = \frac{2}{\sqrt{20}}$



∴ 2nd quadrant.

∴ related angle $\alpha = 26.565°$
$\approx 27°$

∴ Actual angle $\alpha = 180° - 27°$
$\alpha = 153°$

(B)

Section 2

Question 6:

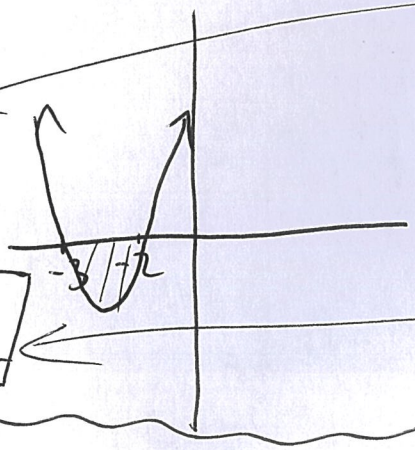
a) $\frac{2x+5}{x+3} < 1$, $x \neq -3$

$(2x+5)(x+3) < (x+3)^2$ ← (1)

$2x^2+11x+15 < x^2+6x+9$

$x^2+5x+6 < 0$ ← (1)

$(x+3)(x+2) < 0$



$\therefore -3 < x < -2$ ← (1)

- Done well
- Still some confusion, however with finding the final inequality

b) ${}^8C_3 \times {}^7C_4 = 1960$ total ways

${}^7C_2 \times {}^6C_3 = 420$ ways if they are on the committee ← (1)

∴ Not to be on the same committee is $1960 - 420 = 1540$ ways. ← (1)

- Around half of the students got this correct
- Need to remember & use complimentary events

Question 6

c) $P(x) = x^4 + px^3 + qx^2 - 5x + 1$

$P'(x) = 4x^3 + 3px^2 + 2qx - 5$

$P'(1) = 0$

$0 = 4 + 3p + 2q - 5$

$0 = 3p + 2q - 1$ ①

$P(1) = 0$

$0 = 1 + p + q - 5 + 1$

$0 = p + q - 3$ ②

① ← 1

② ← 1

② × 2 $0 = 2p + 2q - 6$ ③

① $0 = 3p + 2q - 1$

① − ③ $0 = p + 5$

$\boxed{p = -5}$

Sub into ②

$0 = -5 + q - 3$

$\boxed{q = 8}$

1 both correct

* Mostly done well
* Some were using sum & product of roots then got confused.

d) i) $(1+x)(1+x)^9 = (1+x)\left({}^9C_0 + {}^9C_1x + \dots + {}^9C_{r-1}x^{r-1} + {}^9C_rx^r + \dots + {}^9C_9x^9\right)$

∴ Coefficient of $x^r = \boxed{{}^9C_r + {}^9C_{r-1}}$

① - expansion

① - answer

* Many jumped straight to ${}^{10}C_r$ but I wanted the expansion & the coeffecients, took off a mark.

ii) $(1+x)(1+x)^9 = (1+x)^{10}$

Coefficient of $x^r = {}^{10}C_r$

∴ ${}^{10}C_r = {}^9C_r + {}^9C_{r-1}$

or $\binom{10}{r} = \binom{9}{r} + \binom{9}{r-1}$ as required

* Most used the combinations formula to try and prove the relationship. This was a lot for 1 mark.
  ⇦ Think of easier ways.

① - correctly showing solution

Question 7

a) $4\cos 5x\cos 2x\sin x = \sin 8x - \sin 6x + \sin 4x - \sin 2x$

$LHS = 4\cos 5x\cos 2x\sin x$

$= 2\times 2\cos 5x\cos 2x\sin x$ ← (1)

$= 2(\cos 3x + \cos 7x)\sin x$ ← (1)

$= 2\cos 3x\sin x + 2\cos 7x\sin x$

$= \sin 4x - \sin 2x + \sin 8x - \sin 6x$ ← (1)

$= \sin 8x - \sin 6x + \sin 4x - \sin 2x$

$= RHS$

try to write $2\cos 5\alpha\cos 2\alpha$ $= \cos 7\alpha + \cos 3\alpha$

Instead of $2\cos 5\alpha\cos 2\alpha = 2\times\frac{1}{2}(\cos 7\alpha + \cos 3\alpha)$

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b) $4\sin 2\theta - \tan\theta + 3\cos 2\theta = 3 \quad 0^\circ \le \theta \le 360^\circ$

$4\left(\frac{2t}{1+t^2}\right) - t + 3\left(\frac{1-t^2}{1+t^2}\right) = 3$ ← (1)

$\frac{8t - t(1+t^2) + 3(1-t^2)}{1+t^2} = 3$

$t^3 + 6t^2 - 7t = 0$

$t(t^2 + 6t - 7) = 0$

$t(t+7)(t-1) = 0$ ← (1)

$\therefore t = 0, -7, 1$

For $t=0$

$\tan\theta = 0$

$\theta = 0^\circ, 180^\circ, 360^\circ$

Check $\theta = 180^\circ$

$4\times\sin 360 - \tan 180 + 3\cos 360 = 3$

$3 = 3$ ✓

$\therefore \theta = 180^\circ$ is a solution

$t=1$

$\tan\theta = 1$

$\theta = 45^\circ, 225^\circ$

$t=-7$

$\tan\theta = -7$

$\theta \approx 98^\circ, 278^\circ$

+	+
+	−

−	−
+	−

$\therefore \theta = 0^\circ, 45^\circ, 98^\circ, 180^\circ, 225^\circ, 278^\circ, 360^\circ$

(2) −1 for each error

Many students check for $\theta = 90^\circ$

7c) i) $\cos 3x = 4\cos^3 x - 3\cos x$

LHS $= \cos 3x$

$= \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$ ← (1)

$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \cdot \sin x$ ← (1)

$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$

$= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x$ ← (1)

$= RHS$

Some students didn't show the substitution

$\cos 2x = 2\cos^2 x - 1$ (1)

and $\sin^2 x = 1 - \cos^2 x$

ii) $2\cos x = \sqrt{\cos 3x}$ $(\cos x \geq 0)$

$4\cos^2 x = \cos 3x$

$4\cos^2 x = 4\cos^3 x - 3\cos x$ *using part (i) ← (1)

$0 = 4\cos^3 x - 4\cos^2 x - 3\cos x$

$\cos x(4\cos^2 x - 4\cos x - 3) = 0$

$\cos x(2\cos x - 3)(2\cos x + 1) = 0$ ← (1)

The following answer is given full mark.

$\therefore \cos x = 0, \frac{3}{2}$ and $-\frac{1}{2}$

($\frac{3}{2}$: no solutions, undefined) ($-\frac{1}{2}$: $\times$ $\cos x \geq 0$ ∴ not included)

$\cos x = 0$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$ ← (1)

$\cos x = 0, \frac{3}{2}, -\frac{1}{2}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{2\pi}{3}, \frac{4\pi}{3}$

$x \neq \frac{3}{2}$ $(-1 \leq \cos x \leq 1)$

Question 8

a) i) $p = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $q = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$

$|p - q| = \left| \begin{pmatrix} 5 \\ -3 \end{pmatrix} \right|$ ← (1)

$= \sqrt{5^2 + (-3)^2}$

$= \sqrt{34}$ ← (1)

Done well.

ii) $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + 2\begin{pmatrix} -2 \\ 4 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = 0$

$\underset{\sim}{r}$

$\begin{pmatrix} x - 1 \\ y + 9 \end{pmatrix} = 0$ ← (1)

Done well.

$\underset{\sim}{r} = \begin{pmatrix} x \\ y \end{pmatrix}$

$\underset{\sim}{r} = \begin{pmatrix} 1 \\ -9 \end{pmatrix}$ ← (1)

Q8 b) $\underset{\sim}{u} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\underset{\sim}{v} = \begin{pmatrix} -5 \\ k \end{pmatrix}$

i) $\underset{\sim}{u} \cdot \underset{\sim}{v} = 0$

$\begin{pmatrix} 3 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ k \end{pmatrix} = 0$

$-15 + 4k = 0$

$\boxed{k = \frac{15}{4}}$ ← (1)

Done well.

ii) $\underset{\sim}{v} = -\lambda \underset{\sim}{u}$

$\begin{pmatrix} -5 \\ k \end{pmatrix} = -\lambda \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\therefore -5 = -3\lambda$

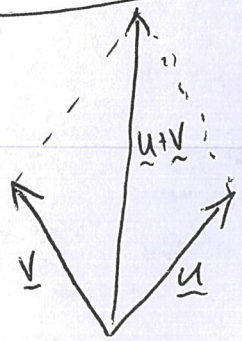
$\boxed{\frac{5}{3} = \lambda}$ ← (1)

$\therefore k = -4\lambda$

$k = -4 \times \frac{5}{3}$

$\boxed{k = -\frac{20}{3}}$ ← (1)

* Some students had trouble with this question.
* Some wrote $\underset{\sim}{u} = -\underset{\sim}{v}$ and then find the value of k. 1 mark deducted for that.
* students need to show proper working out.
* students used different methods to solve this problem.

iii) 

We need $|\underset{\sim}{u}| = |\underset{\sim}{v}|$

$\sqrt{3^2 + 4^2} = \sqrt{(-5)^2 + k^2}$ ← (1)

$\sqrt{25} = \sqrt{25 + k^2}$

$25 = 25 + k^2$

$0 = k^2$

$k = 0$ ← (1)

Some students had trouble with this but mostly done well.

Some used the result $(\underset{\sim}{u} + \underset{\sim}{v}) \cdot (\underset{\sim}{u} - \underset{\sim}{v}) = 0$ to find k, which is correct.

Question 9

a) $2^n - 1$, divisible by 3 for all even positive integers

Prove true for $n = 2$

$2^2 - 1 = 4 - 1$

$= 3$ which is divisible by 3

$\therefore$ true for $n = 2$ (1)

Assume true for $n = 2K$, K is an integer

$2^{2K} - 1 = 3M$

$\therefore 2^{2K} = 3M + 1$ * (1)

Prove true for $n = 2K+2$ i.e. $2^{2k+2} = 3N$, N an integer.

$2^{2K+2} - 1 = 2^2 \times 2^{2K} - 1$

$= 4 \times 2^{2K} - 1$ (1)

$= 4(3M+1) - 1$ using assumption *

$= 12M + 4 - 1$

$= 12M + 3$

$= 3(4M+1)$

which is divisible by 3 (1)

$\therefore$ true for $n = 2K+2$

Hence, by Mathematical induction the statement is true for all even positive integers

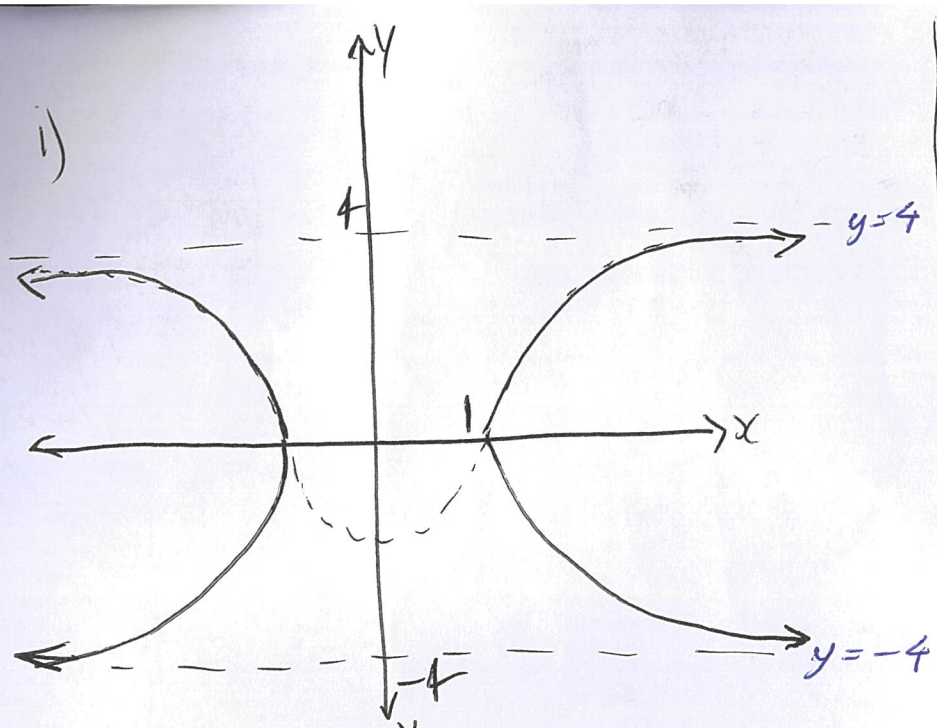
Mainly well done

→ Some students did 2^k, keeven which is OK but can be harder to prove in some questions. Recommend to use 2^{2k}.

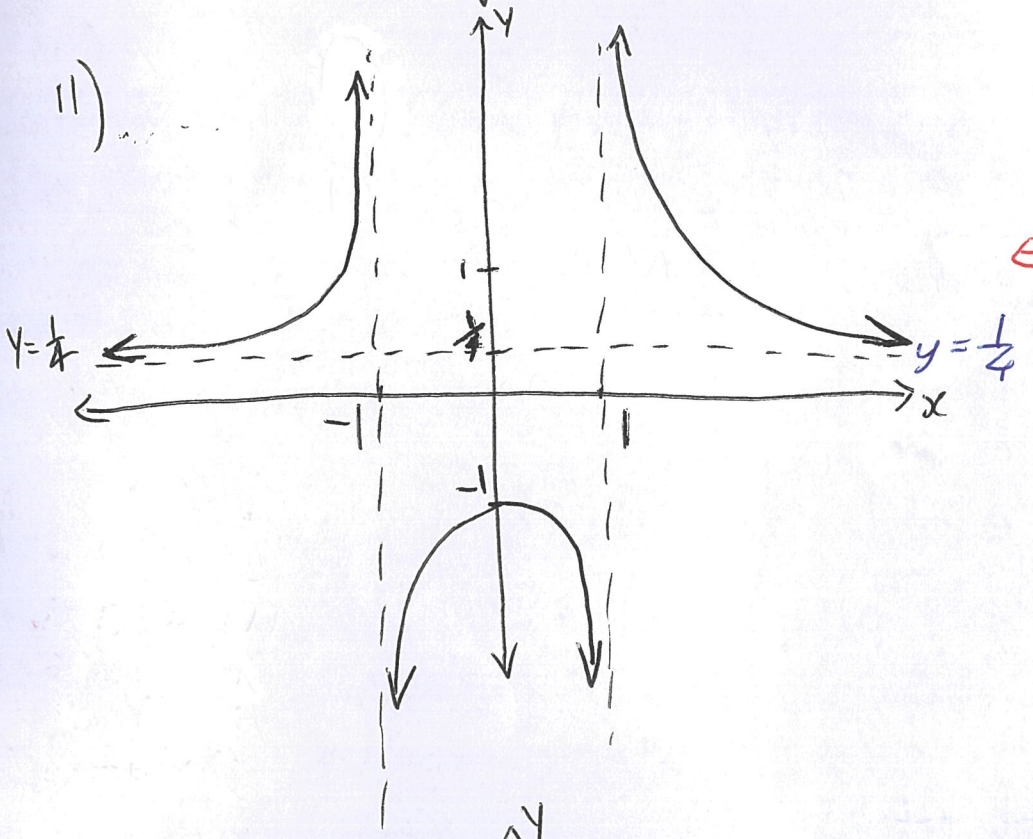
← Some students missed this line.

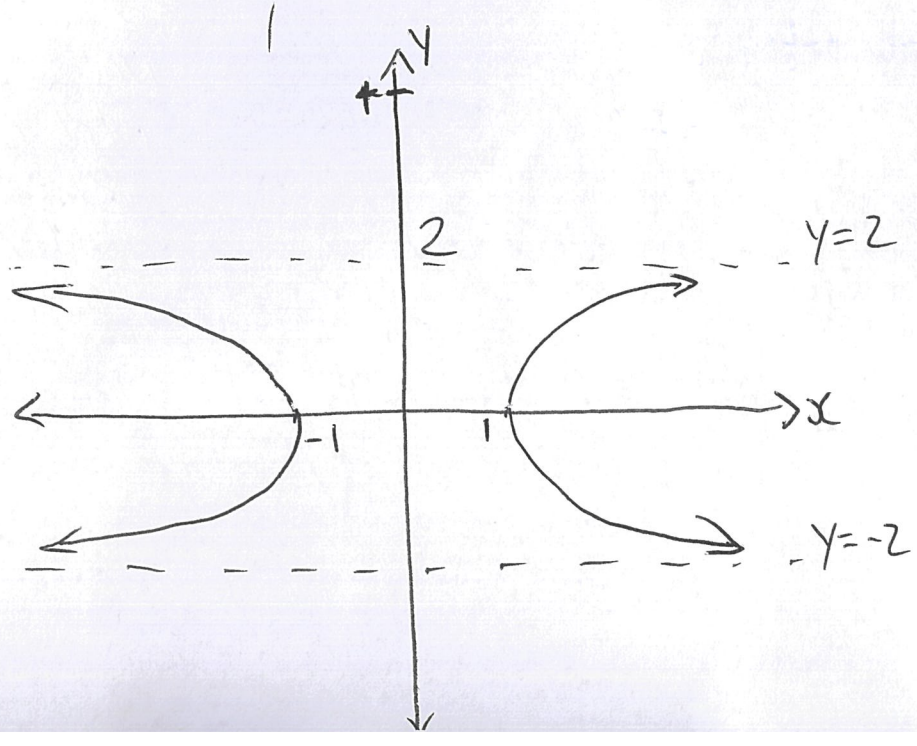
Note: Some students tried to prove true for 2^{k+1} or 2^{2k+1} NOT 2^{2k+2} which is INVALID & meant they could get no further marks.

9b) i)



ii)





2 each
- correct shape & asymptotes & intercepts etc

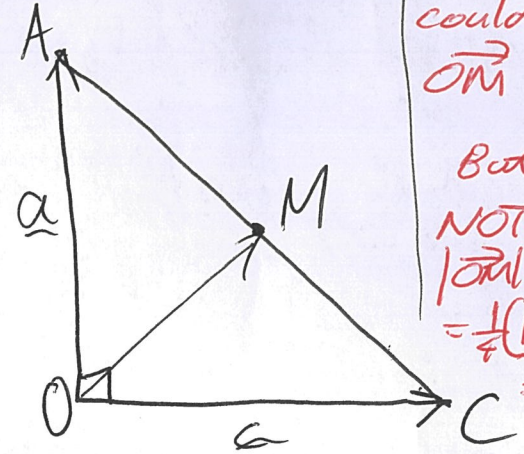
Some students drew (b)(i) as though it was $y = |f(x)|$ not $|y| = f(x)$. Asymptotes & reflections in the x axis were often missed.

(ii) Mainly well done, although some students missed the horizontal asymptote at $y = \frac{1}{4}$.

Once again, many students forgot the reflection in the x axis & the asymptotes.

9c) Since M is the midpoint of AC, it is equidistant from A & C already

To prove: $|\overrightarrow{OM}| = \frac{1}{2}|\overrightarrow{AC}|$



$$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AC}$$
$$= \underset{\sim}{a} + \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})$$
$$= \frac{1}{2}(\underset{\sim}{c} + \underset{\sim}{a})$$

$$|\overrightarrow{OM}| = \frac{1}{4}|\underset{\sim}{c} + \underset{\sim}{a}|^2$$
$$= \frac{1}{4}\left[(\underset{\sim}{c} + \underset{\sim}{a}) \cdot (\underset{\sim}{c} + \underset{\sim}{a})\right]$$
$$= \frac{1}{4}\left[\underset{\sim}{c}\cdot\underset{\sim}{c} + 2(\underset{\sim}{a}\cdot\underset{\sim}{c}) + \underset{\sim}{a}\cdot\underset{\sim}{a}\right]$$
$$= \frac{1}{4}\left(\underset{\sim}{c}\cdot\underset{\sim}{c} + a\cdot a\right) \quad \text{as } \underset{\sim}{a}\cdot\underset{\sim}{c} = 0 \text{ as } \underset{\sim}{a} \perp \underset{\sim}{c}$$
$$= \frac{1}{4}\left(|\underset{\sim}{c}|^2 + |\underset{\sim}{a}|^2\right)$$

Generally students could get $\overrightarrow{OM} = \frac{1}{2}\underset{\sim}{c} + \underset{\sim}{a}$.

But they could NOT get $|\overrightarrow{OM}|^2 = \frac{1}{2}(\underset{\sim}{c}+\underset{\sim}{a})\cdot\frac{1}{2}(\underset{\sim}{c}+\underset{\sim}{a})$
$= \frac{1}{4}[|\underset{\sim}{c}|^2 + 2\underset{\sim}{c}\cdot\underset{\sim}{a} + |\underset{\sim}{a}|^2]$
$= \frac{1}{4}(|\underset{\sim}{c}|^2 + |\underset{\sim}{a}|^2)$.

① - $\overrightarrow{OM}$ or $\overrightarrow{AM}$ or $\overrightarrow{CM}$

① - $|\overrightarrow{OM}|$

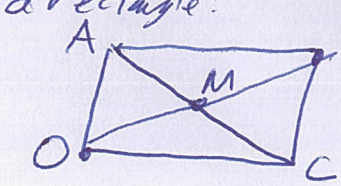
A lot of students treated $\underset{\sim}{a}$ & $\underset{\sim}{c}$ as though they were LENGTHS not vectors & used Pythagoras. It WAS possible to do this using Pythagoras, but the ① - final solution misconception that $\underset{\sim}{a}$ & $\underset{\sim}{c}$ were lengths & not vectors led to a lot of students losing marks.

COULD do $|\overrightarrow{AC}|^2 = |\underset{\sim}{a}|^2 + |\underset{\sim}{c}|^2$
$|\overrightarrow{AM}|^2 = \frac{1}{2}(|\underset{\sim}{a}|^2 + |\underset{\sim}{c}|^2)$
As $\overrightarrow{OM} = \frac{1}{2}(\underset{\sim}{a}+\underset{\sim}{c})$ & $\underset{\sim}{c} \perp \underset{\sim}{a}$ $|\overrightarrow{OM}|^2 = \frac{1}{4}[|\underset{\sim}{a}|^2 + |\underset{\sim}{c}|^2]$

$\overrightarrow{AM} = \frac{1}{2}(\underset{\sim}{c} - \underset{\sim}{a})$.
$|\overrightarrow{AM}|^2 = \frac{1}{2}(\underset{\sim}{c}-\underset{\sim}{a})\cdot\frac{1}{2}(\underset{\sim}{c}-\underset{\sim}{a})$
$= \frac{1}{4}[|\underset{\sim}{c}|^2 - 2\underset{\sim}{a}\cdot\underset{\sim}{c} + |\underset{\sim}{a}|^2]$
$= \frac{1}{4}[|\underset{\sim}{c}|^2 + |\underset{\sim}{a}|^2]$
$= |\overrightarrow{OM}|^2$.

Hence As $|\overrightarrow{AM}| = |\overrightarrow{MC}|$ & $|\overrightarrow{AM}| = |\overrightarrow{OM}|$
M is equidistant from O, A & C.

One alternative was to complete a rectangle:



& use the fact that rectangle diagonals are equal & bisect each other.

but if you just wrote $\underset{\sim}{a}$ & $\underset{\sim}{c}$ you didn't get the marks. You need to use LENGTHS.

Question 10

a) $\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \cos 45^\circ$

$$ac+bd = \frac{1}{\sqrt{2}}\sqrt{(a^2+b^2)(c^2+d^2)}$$

$$(ac+bd)^2 = \frac{1}{2}(a^2+b^2)(c^2+d^2)$$ ← (1)

Most students got the first mark, but didn't progress far enough to get the second mark.

(i) $2(ac+bd)^2 = a^2c^2+a^2d^2+b^2c^2+b^2d^2$

Now completing the squares

$$2(ac+bd)^2 = b^2c^2-2abcd+ad^2+ac^2+2abcd+b^2d^2$$ ← (1)

NOTE: SINCE THIS IS A "SHOW" QUESTION YOU CAN'T USE THE RESULT TO PROVE THE RESULT! Any attempt using this obviously got 0 mark.

$$2(ac+bd)^2 = (bc-ad)^2+(ac+bd)^2$$

$$(ac+bd)^2 = (bc-ad)^2$$ ← (1)

ALTERNATIVE METHOD:

From (1)

$2a^2c^2+4abcd+2b^2d^2 = a^2c^2+a^2d^2+b^2c^2+b^2d^2$

$-a^2c^2-b^2d^2-2abcd$ from BS:

$a^2c^2+2abcd+b^2d^2 = a^2d^2-2abcd+b^2c^2$ ← (1)

$(ac+bd)^2 = (ad-bc)^2$ Same.

or $(ac+bd)^2 = (bc-ad)^2$ → (1)

Alternative method was used by most successful students.

Q10b)

i) show $(x^2+2x-1)(x+2) = x^3+4x^2+3x-2$

$LHS = x^3+2x^2+2x^2+4x-x-2$

$= x^3+4x^2+3x-2$

$= RHS$ as required

ii) Prove $0\times1!+7\times2!+26\times3!+\dots+(n^3-1)\times n! = (n^2-2)(n+1)!+2$

Prove true for $n=1$

$LHS = 0\times1!$ $RHS = (1^2-2)(1+1)!+2$

$=0$ $= -1\times2!+2$

$=0$

$\therefore$ True for $n=1$

Assume true for $n=k$

$0\times1!+7\times2!+\dots+(k^3-1)k! = (k^2-2)(k+1)!+2$

Prove true for $n=k+1$

$0\times1!+\dots+(k^3-1)(k!)+((k+1)^3-1)\times(k+1)! = ((k+1)^2-2)(k+1+1)!+2$

$= (k^2+2k-1)(k+2)!+2$

$LHS = 0\times1!+\dots+(k^3-1)k!+((k+1)^3-1)\times(k+1)!$

$= (k^2-2)(k+1)!+2+((k+1)^3-1)(k+1)!$ * using assumption for $n=k$

$= (k+1)!(k^2-2+k^3+3k^2+3k+1-1)+2$

$= (k+1)!(k^3+4k^2+3k-2)+2$

$= (k+1)!(k+2)(k^2+2k-1)+2$ * using part (i)

$= (k^2+2k-1)(k+2)(k+1)!+2$

$= (k^2+2k-1)(k+2)!+2$

$= RHS$

$\therefore$ true for $n=k+1$

$\therefore$ The statement is proved true by Mathematical induction

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Very well done.

(1) - correct solution

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(1) Common mistakes: Split your proof into LHS & RHS.

(1) You CAN'T HAVE THEM TOGETHER OR YOU'RE ASSUMING WHAT YOU TO PROVE!

(1) Not stating "By assumption" or similar lost a mark.

(1) Some students used other methods than "hence" to factorise k^3+4k^2+3k-2. You were told HENCE; not HENCE OR OTHERWISE, so 1 mark was deducted if you did this.

Careless errors which meant the result could no longer be proven meant NO error carried forward from that point. (as you could not get to the proof).

Question 11

a) i) $\dfrac{(2x\cos 2x)^3}{8\sin^3 2x} = \dfrac{8x^3\cos^3 2x}{8\sin^3 2x}$

$= \dfrac{x^3}{\tan^3 2x}$ or $x^3\cot^3 2x$

(1) - correct

Generally well done.

ii) $\lim\limits_{x\to 0} \dfrac{2x^3}{\tan^3 2x} = 2\lim\limits_{x\to 0} \dfrac{x^3}{\tan^3 2x}$

$= 2\times\lim\limits_{x\to 0} \dfrac{(2x\cos 2x)^3}{8\sin^3 2x}$ *from part (i)

(1) ← A lot of students got to this stage but had difficulties going further.

$= \dfrac{2}{8}\lim\limits_{x\to 0} \dfrac{(2x)^3\cos^3 2x}{\sin^3 2x}$

recognise & use the rule $\lim\limits_{x\to 0}\dfrac{x}{\sin x} = 1$ →

$= \dfrac{1}{4}\lim\limits_{x\to 0} \dfrac{(2x)^3}{\sin^3 2x} \times \cos^3 2x$

$= \dfrac{1}{4}\times 1 \times \lim\limits_{x\to 0}\cos^3 2x$

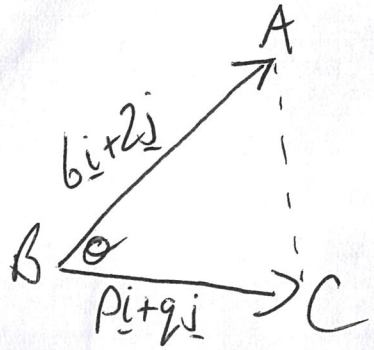
(1) ←

$= \dfrac{1}{4}\times 1$

$= \dfrac{1}{4}$

(1) ←

Q11 b)



$|\vec{BC}| = \sqrt{p^2+q^2}$ $\quad |\vec{BA}| = \sqrt{6^2+2^2} = \sqrt{40}$

$(8\sqrt{5})^2 = p^2+q^2$

$320 = p^2+q^2$ (1)

$\text{Area}_{\triangle ABC} = \frac{1}{2} \times |\vec{BC}||\vec{BA}| \sin\theta$

$40 = \frac{1}{2} \times \sqrt{p^2+q^2} \times \sqrt{40} \times \sin\theta$

$80 = \sqrt{320} \times \sqrt{40} \times \sin\theta$

$\sin\theta = \frac{80}{\sqrt{320}\times\sqrt{40}}$

$\sin\theta = \frac{1}{\sqrt{2}}$

$\therefore \theta = 45^\circ$ (as θ is acute)

$\vec{BA} \cdot \vec{BC} = |\vec{BA}| \cdot |\vec{BC}| \cdot \cos 45$

$\begin{pmatrix} 6 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} p \\ q \end{pmatrix} = \sqrt{40} \times \sqrt{320} \times \frac{1}{\sqrt{2}}$

$6p+2q = 80$

$3p+q = 40$

$\boxed{q = 40-3p}$ (2)

Sub into $p^2+q^2=320$

$p^2+(40-3p)^2 = 320$

$10p^2-240p+1280 = 0$

$p^2-24p+128 = 0$

$(p-8)(p-16) = 0$

$\boxed{p = 8 \text{ and } 16}$

When $p=8$: $q = 40-3\times 8$, $q = 16$

When $p=16$: $q = 40-3\times 16$, $q = -8$

$\therefore p=8, q=16$ and $p=16, q=-8$

Comments/Feedback

(1) 1, Find $|\vec{BC}|$ and $|\vec{BA}|$

(1) 2, Use $A = \frac{1}{2}ab\sin C$ to find θ

(1) 3, Use dot product to create a relationship for p and q

(2) 4, Solve simultaneously.

- A lot of students had difficulty with this question

- A large number did not know where to begin

Advice:

Begin a question of this nature by drawing a diagram, marking all the information on the diagram and seeing what that information will allow you to find ⇒ which will eventually lead you to the answer.